B.Sc Part I (Hons)
Paper II (Differential [redacted] Calculus.

Successive differentiation:—

1. nth differential of x^m

$$\text{Let } y = x^m$$

Then
$$y_1 = m x^{m-1}$$
$$y_2 = m(m-1)\, x^{m-2}$$
$$y_3 = m(m-1)(m-2)\, x^{m-3}$$
$$-- - - - - ---$$

Let us suppose that
$$y_n = m(m-1)(m-2)(m-3) -- (m-n+1)\, x^{m-n} \quad \text{—(1)}$$

Differentiating again, we get
$$y_{n+1} = m(m-1)(m-2)(m-3) -- (m-n+1)(m-n)\, x^{m-n-1}.$$

Which is of the same form as (1) with n replaced by $(n+1)$.

Since (1) is true for $n = 1, 2, 3$, therefore by the method of induction, (1) is true for all n. That is,

if $y = x^m$, then $y_n = m(m-1)(m-2)(m-3) -- (m-n+1)\, x^{m-n}$

Similarly, if $y = (ax+b)^m$, then $y_n = m(m-1)(m-2) -- (m-n+1)(ax+b)^{m-n}$.

Case 1. If $m = n$, then
$$y_n = n(n-1)(n-2)(n-3) --- (n-n+1)\, x^{n-n}$$
$$= n(n-1)(n-2)(n-3) --- 321 = n!$$

Thus if $y = y^n$, then $y_n = n!$

Hence $y_{n+1} = 0,\ y_{n+2} = 0$ etc.

As a particular case if $y = x^3$, then $y_3 = 3!$

Therefore $y_4 = 0$ etc.

Find the nth differential coefficient of $\frac{1}{ax+b}$.

$$\text{Let } y = \frac{1}{ax+b} = (ax+b)^{-1}$$

$$\therefore y_1 = (-1)(ax+b)^{-2} \times a$$

$$y_2 = a(-1)(-2)(ax+b)^{-3} \times a$$

$$= a^2(-1)(-2)(ax+b)^{-3}$$

$$y_3 = a^2(-1)(-2)(-3)(ax+b)^{-4} \times a$$

$$= a^3(-1)(-2)(-3)(ax+b)^{-4}.$$

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Let us suppose that

$$y_n = a^n(-1)(-2)(-3) \text{ --- to } n \text{ factors} \times (ax+b)^{-n-1}$$

$$= \frac{a^n(-1)^n 1.2.3 \text{ --- } n}{(ax+b)^{n+1}} = \frac{a^n(-1)^n n!}{(ax+b)^{n+1}} \quad \text{--- (2)}$$

Differentiating (2) again with respect to x, we get

$$y_{n+1} = a^n(-1)^n(n!)(-n-1)(ax+b)^{-n-1-1} \times a.$$

$$= a^n(-1)^n(n!)(-1)(n+1) \cdot \frac{a}{(ax+b)^{n+2}}$$

$$= \frac{a^{n+1}(-1)^{n+1}(n+1)!}{(ax+b)^{n+2}}$$

Which is of the same from as (2) with n replaced by $n+1$. Since (2) is true for $n = 1, 2, 3,$ — hence by the method of induction it is true for all n.

Thus if $y = \frac{1}{ax+b}$, then $y_n = \frac{a^n(-1)^n n!}{(ax+b)^{n+1}}$

In particular case if $a = 1$, then $y = \frac{1}{x+b}$

$$y_n = \frac{(-1)^n n!}{(x+b)^{n+1}}.$$

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